Distributed Combustion Effects on Particle Damping

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Virtually all solid propellants contain metal or metallic compounds as additives. A lack of mechanistic understanding is particularly evident in combustion instability work where typical acoustic analyses assume that all particulate combustion occurs at the propellant surface. Although it is widely accepted that this assumption is not physically accurate, there has been little previous effort to mathematically model the interactive effects of distributed combustion with the acoustic wave. This paper presents a model for calculating the effect of distributed combustion on viscous particle damping due to an acoustic wave. The model accounts for the changing particle size of burning aluminum for both original aluminum particles and agglomerated aluminum, burning to either oxide smoke or an oxide cap of specified size. The calculations have been compared to experimental data for a typical aluminized composite propellant. The calculated results show that significant combustion of the metal can occur relatively far from the burning surface and that the assumption that acoustic particle damping is constant for a given frequency is not valid. The calculated particle damping changed by a factor of two over a typical range of conditions used in T-burners.

Introduction

VIRTUALLY all solid propellants contain metal or metallic compounds as additives. Aluminum is added to propellants to increase performance, while a variety of additives are used in reduced-smoke propellants as acoustic stabilizing additives. Both the aluminum and the other additives probably burn in the gas phase after leaving the propellant surface. This mode of combustion can have a significant influence on both combustion efficiency and acoustic-related instability. A lack of mechanistic understanding is particularly evident in combustion instability work where current acoustic analyses[1] assume that all particulate combustion occurs at the propellant surface. However, thermochemical calculations and actual motor data indicate that so-called "inert" suppression additives do burn[2] and that aluminum can burn far from the surfaces.[3-6] The question is, where do they burn and what influence does this have on the acoustics of a rocket motor?

Although it is widely accepted that these assumptions are not accurate, no one has attempted to model the interactive effects of distributed combustion with the acoustic wave. Two effects of distributed combustion can be anticipated. First, there will be changes in the inert, viscous damping of particles as the particle size changes as a function of distance. Second, there will be an effect due to the interaction of the particle combustion and the acoustic wave. Exploring these two approaches appears to have significant potential for resolving some of the questions relating to particle damping effects in motors and laboratory devices such as the T-burner. This paper addresses the first of those two aspects, namely the changes in viscous particle damping due to the changing particle size of burning aluminum particles. It is hoped that future work will also address other additives.

Particle damping due to viscous drag is a well-established phenomenon. Temkin and Dobbins[7] and Carhart and Epstein[8] established the theoretical basis for particle damping, verifying the theory by measuring the damping of aerosols on acoustic waves. More recently, the workers at Naval Weapons Center[9-11] have performed a series of experiments using various burners with propellants containing well-characterized particles. Their measurements before, during, and after combustion compare extremely well with their corresponding calculations, verifying quantitatively particulate damping theory in a combustion environment. Their configuration was that of an end-burning T-burner that would result in relatively large residence times, minimizing the effect of aluminum burning in the burner (the significance of this will be discussed later).

Particle Damping Background

Particulate damping in an acoustic wave has normally been described by the Dobbins and Temkin equation (as modified by Culick[1]),

$$\alpha = \frac{\omega}{2}\frac{C_m}{1+C_m}\frac{\omega\tau}{1+(\omega\tau)^2} \tag{1}$$

where α is the temporal damping coefficient in s^{-1}, ω the angular frequency, C_m the mass fraction of particulate matter, and τ the viscous relaxation time, $\rho D_p^2/18\mu$. The damping is dependent on the frequency of the acoustic wave, the concentration of the particles, and the relaxation time of the particle, which is principally a function of the particle diameter. In Eq. (1), it is usually assumed that particle diameter is constant and uniform throughout the acoustic wave. This is the assumption that is being relaxed in the present work. In the case of aluminized propellants, some of the aluminum (particularly agglomerates) burns in the gas phase relatively far from the burning surface. Also, combustion of relatively large aluminum agglomerates leads to relatively large oxide droplets, so that the droplet size distribution varies with position, reflecting the progress of combustion.

The Dobbins-Temkin equation for the particulate damping in an acoustic medium assumes that the particulates are of a uniform size throughout the acoustic wave. However, for most practical cases one must consider the distribution of different particle types and sizes throughout the acoustic wave. Culick[12] has shown that the viscous particle damping will be proportional to the acoustic velocity squared that is greatest at the center of a burner or rocket motor,

$$\alpha = -\frac{1}{2E^2}\left(\frac{1}{1+\bar{X}}\right)\sum_i \int X_i \phi_i \left(\frac{\nabla\psi}{k}\right)^2 dV \tag{2}$$

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where

$$\phi_i = [\omega(\omega\tau_i)]/[1+(\omega\tau_i)^2]$$

and the summation is performed over all particle sizes, E is the acoustic energy of the wave, and ψ is the mode shape. From Eq. (2), one can expect to get a maximum damping effect due to the particles that exist in the center of the acoustic wave, while those at the ends of the acoustic wave (where the acoustic velocity is less) will have a much smaller effect.

It should be noted that there is an optimum particle size for a given frequency and that there is an optimum frequency for a given particle size (or particle size distribution). For very large or very small particle sizes, a very low particle damping occurs with a maximum somewhere between. In addition to the inherent optimum conditions due to frequency and particle diameter, one also must consider the superposition of residence time due to the velocity of the gases carrying the particles through the cavity. Thus, depending on the velocity of the gases, an optimum damping condition can also occur strictly due to the fact that the reference time of the particles in the cavity is small or large.

Aluminum Agglomeration and Combustion

For typical solid propellants, the size of the aluminum agglomerate coming off of the surface of the burning propellant will depend on many factors, including the size of the oxidizer, the burning rate of the propellant, the pressure level of the rocket motor, and the cross-flow velocity over the surface of the propellant.[3-6] Typically, the aluminum will leave the surface in a bimodal distribution. Parent aluminum particles will emerge from the burning surface and enter into the gas phase at their original size. Also some aluminum particles will agglomerate into a much larger size and enter the gas phase as such. A conceptual diagram of what occurs is contained in Fig. 1. The length of time that these two types of aluminum particles persist in the gas phase depends on the size of the individual particle or agglomerate as it leaves the surface. A larger agglomerate will reside in the gas phase for a much longer time duration than a smaller parent particle. Thus, it is apparent that there are at least two size distributions of burning aluminum that can occur in solid-propellant rocket motor environments and that will ultimately have an influence on the particulate damping in an acoustic wave.

The combustion of the aluminum can be estimated using what is referred to as a D^2 law (e.g., Ref. 5). A D^2 law accounts for the rate of change of the burning diameter of the aluminum particle as a function of time and is expressed as

$$D_p^n = D_0^n - \beta t \tag{3}$$

where the exponent n is usually assumed to be ~ 1.5-2.0 and β is a coefficient of burning for aluminum having the form[5] $\beta_0 + bP$. Equation (3) shows that the combustion of the aluminum normally will take place over a considerable period of time, depending on the size of the aluminum particle for typical values of β ($\sim$200-400 $\mu m^2/ms$).[5] For the calculations made in this paper, n was taken as 2 with a β_0 of 180 $\mu m^2/ms$ and b of 64.7 $\mu m^2/ms/MPa$.[5]

To determine the effects due to changing particle size as the aluminum particles burn, one must compare the particle combustion time to the characteristic flow time of the environment, thus determining an overall residence time for the burning particles. The fact that the aluminum particles are changing size as they pass through the acoustic cavity can and does have a significant effect on particulate damping. Parametric calculations comparing results from Eq. (3) with residence time calculations show that for many very typical situations, an aluminum agglomerate typical of a composite solid propellant will not have a sufficiently long residence time for it to be totally consumed within a typical rocket motor cavity. Figure 2 contains such a comparison for an 800 Hz cavity at 3.4 MPa (500 psi) and a propellant burn rate of 0.81 cm/s (0.32 in./s). The figure shows that agglomerates of 80 μm or greater will not burn completely before leaving the cavity for a propellant area ratio of 6 or greater. This means that the aluminum agglomerate will be continually changing size as it passes through the cavity, having a different effect on particulate damping as it does so.

Conditions with long residence times lead to situations where the aluminum could burn completely to aluminum oxide smoke. Long residence times correspond to low frequencies, high pressures, very-low-burn-rate propellants, and configurations that lead to low gas velocities. Higher frequencies, lower pressures, or high-burn-rate propellants result in high gas velocities and short residence times. Aluminum agglomerates will seldom burn completely for a short residence time condition. However, most unagglomerated particles (on the order of 20 μm or less) will be totally consumed before they leave the cavity. Thus, one has to take into account and consider both the combustion of agglomerates and the combustion of the smaller parent particles in a different manner, due to the fact that the one may not be consumed, whereas the other probably will.

Modifications and Use of the SSP Program

To provide a basis for the current work, the standard stability program (SSP) developed by the Air Force for analyzing both motor and T-burner acoustics[13] has been used as a convenient tool. The SSP provides a convenient basis where the acoustic equations have already been programmed and incorporated into a computer program facilitating parametric studies. The effect of particulate damping on acoustics for either a rocket motor or a T-burner cavity is available within the program. However, to accommodate changing particle sizes, Eqs. (2) and (3) have been programmed along with several other modifications to the SSP computer program.

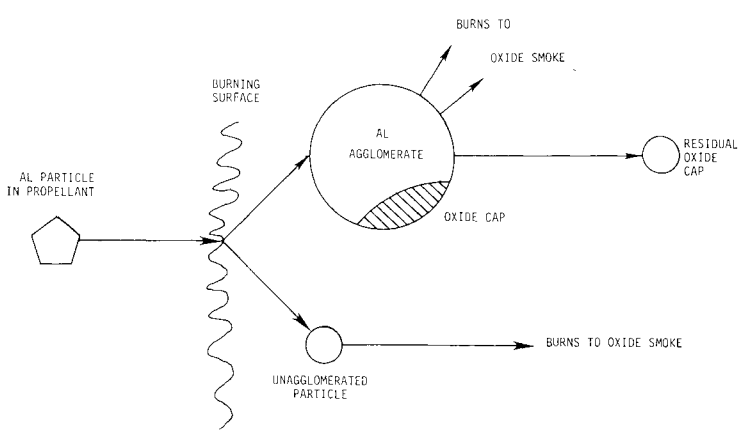

Fig. 1 Conceptual mechanisms of aluminum combustion in propellants.

The original SSP program allows for a distribution of particle sizes, but that distribution of particle sizes must remain constant throughout the acoustic cavity. Therefore, the program has been modified to allow for varying particle size distributions throughout the length of the acoustic cavity. Modifications have been made that allow for the four different particle types shown in Fig. 1. These correspond to a parent aluminum particle, an aluminum agglomerate, aluminum oxide smoke, and a residual aluminum oxide cap. The two different size aluminum particles are coupled to a burning rate law, which allows those particles to change size as a function of time based on Eq. (3). The time is related to the distance traveled and the velocity distribution in the cavity. Parent aluminum particles burn directly to aluminum oxide smoke, while an agglomerate burns to oxide smoke until it ultimately forms an oxide cap. An aluminum oxide cap is assumed to form from each aluminum agglomerate. The program allows for an input percentage of the aluminum to be agglomerated and each of the aluminum agglomerates burns to form an oxide cap. As an agglomerate burns, the product is assumed to be aluminum oxide smoke. The size of the aluminum oxide smoke is a fixed input value, as is the oxide cap size. At each segment of the cavity, the amount of aluminum that would burn is calculated, the new aluminum particle size is introduced, and the corresponding amount of aluminum oxide smoke is introduced. When a burning agglomerate reaches the size of a cap, it is then given oxide properties and its size fixed. These parameters can be input conveniently and the program proceeds to make the appropriate size and concentration calculations for the four different types of particles throughout the length of the burner.

Comparison to ANB-3066 T-Burner Data

A quantitative comparison has been made between the model and actual T-burner data obtained from Ref. 14 for ANB-3066 propellant. ANB-3066 was the standard propellant used as part of the T-burner workshop program[14-18] that was performed in the early 1970s. It is a typical composite propellant with a moderate burning rate and a relatively high percentage of aluminum. Data obtained from the variable area T-burner on that program indicated a particulate damping level on the order of 75 s^{-1} for a frequency of 800 Hz and a pressure of 34 MPa (500 psi). However, the pulsed T-burner gave a particle damping value of ~ 125 s^{-1}. The discrepancy in the two numbers has never been fully resolved. It is the authors' opinion that this discrepancy is due to two factors: 1) the differing propellant orientation, perpendicular to the flow vs parallel to the flow, will influence the size of the agglomerate; and 2) the pulsed T-burner has a very long residence time due to the small amount of propellant used, whereas the variable area T-burner results in much shorter and variable residence times leading to a distributed combustion effect. This second factor is the subject of the following paragraphs.

In order to pursue the approach used, the size and fraction the aluminum agglomerates must be known. Although there is no method available for predicting the amount of aluminum that agglomerates, measurements[4,5] indicate that the percentage agglomeration for a typical, high-solids composite propellant is about 10-25%. Worse cases can be cited, but for the propellant such as ANB-3066, agglomeration percentages of 15-25% and an agglomerate size of 100 μm appear reasonable based on data[4] and using the correlations of Ref. 19.

Using the model described above, damping values were calculated to try to determine what combination of agglomerate size, fraction of aluminum agglomerated, and smoke size would give damping values matching the data from the T-burner testing. The calculated results are plotted in Figs. 3 and 4 where it is apparent that the damping is very dependent upon the assumed size of the aluminum oxide

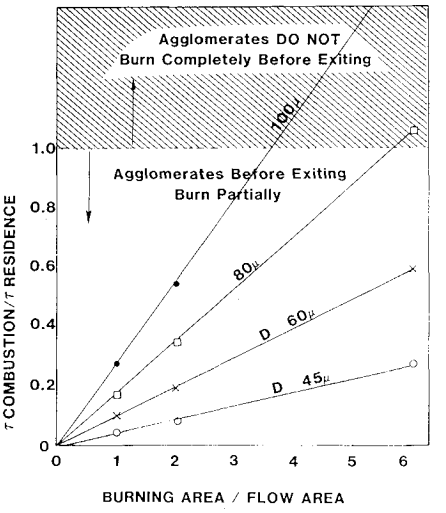

Fig. 2 Comparison of aluminum agglomerate combustion and residence times for an 800 Hz burner at 3.4 MPa (500 psi) and 0.81 cm/s (0.32 in./s) varying the relative amount of propellant.

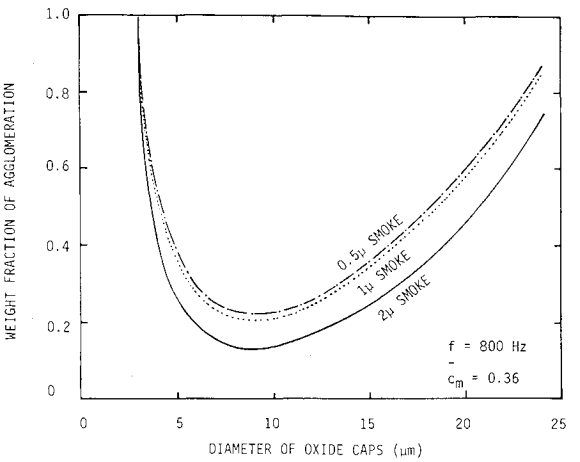

Fig. 3 Parametric curves showing conditions required to produce 75 s^{-1} of damping at 800 Hz corresponding to variable area T-burner data.

smoke, the fraction of aluminum agglomerated, and the size of the residual oxide cap. The calculations showed little damping due to the original aluminum agglomerate. A nominal agglomerate size of 100 μm was assumed. Variations in the agglomerate size influence only the calculated damping through the residence time effect of the agglomerate. Assuming an aluminum oxide smoke of 1 μm or less results in insufficient damping for the test cases considered. The desired value of damping could not be achieved unless a relatively large fraction (25%) or near optimal diameter particles (i.e., oxide caps) were present in addition to the 1 or 2 μm smoke. Optimal diameter for the given conditions was determined to be approximately 9 μm based on calculations from Eq. (1). Therefore, the parametric calculations shown in Figs. 3 and 4 were performed for smoke sizes of 0.5, 1, and 2 μm, assuming different particle sizes and amounts of agglomerated aluminum. Figure 3 is for the combination of parameters needed to produce the -75 s^{-1} of damping corresponding to data from the variable area T-burner. Figure 4 is the same thing for the -125 s^{-1} corresponding to the longer residence time, pulsed T-burner. Workers at NWC[9-11] have made particle size distribution measurements from T-burners and residue bombs to provide input to Eq. (1). Their calculated damping values compare very favorably to measurements made in their pulsed T-burner. Their particulate measurements indicate a distribution of ~ 0.5-3 μm as the

principal smoke component, but they also measured a significant fraction of oxide ≥ 10 μm. The fact that they observed good agreement between measurements and theory is consistent with the relatively long residence times of the pulsed T-burner; the aluminum apparently burns close to the surface relative to the acoustic wave.

Although the results indicate that there is a large combination of conditions that could lead to the observed damping value, many of those conditions are not realistic. Four conditions were selected that appear to be realistic involving either 1 or 2 μm smoke. Those four conditions are tabulated in Table 1 and form the basis for the parametric calculations discussed in the next section. The experimental test conditions[14-18] are also tabulated in Table 1.

Results

The model was used to calculate the particle damping varying the amount of propellant in the T-burner to simulate the variable area T-burner experiments. Figure 5 shows the calculated damping coefficient plotted vs the area ratio (i.e., the burning area of propellant divided by the burner cross-sectional area) and all four test cases of Table 1a. The four curves are very similar. Approaching zero propellant in the burner (long residence time), the damping asymptotically approaches a value of $\sim 80\ s^{-1}$. As the amount of propellant increases and the residence time decreases, the damping decreases dramatically approaching limiting values of $\sim$30-40 s^{-1} at the largest area ratio for which experimental data exists. The decrease in damping is due to the fact that the aluminum agglomerates are being swept out of the burner without fully burning. They are sufficiently large that their contribution to the overall damping is negligible. The oxide caps that form for long residence times contribute $\sim 40\ s^{-1}$, but for short residence times they do not have a chance to form. These calculations indicate that for the assumed particle configurations, the particle damping will change by a factor of at least two. Therefore, in T-burner data reduction or in extrapolating to motor conditions, the standard assumption of taking particle sizes as constant can be expected to introduce very large errors. Obviously, the calculations that have been made here are subject to the accuracy of the assumed particle size distributions. However, for the four test cases, which cover a fairly wide range of conditions, the final results are very similar—qualitatively showing the same effect that is very dependent on the burning of the aluminum to an oxide cap. This is an indirect verification that the results are at least qualitatively correct.

Particle damping calculations were also made varying pressure and frequency corresponding to the original test conditions as outlined in Table 1b. Figure 6 shows the calculated damping plotted vs pressure for cases II and III, each at area ratios of 1½ and 6. The results are very similar to Fig. 5. The effect of pressure is seen to be very small. This is to be expected as pressure does not enter directly into Eq. (1). The influence of pressure is manifest through the gas velocity, which is inversely proportional to density (i.e., pressure). At low pressures the gas velocity increases, the residence time decreases, and the damping decreases. Apparently the longer residence time allows the more efficient large particles to burn down to the smaller less efficient sized smoke, thus reducing the overall damping. This is consistent with the effect of larger area ratios, which also decreases the residence time. The net result is reduced damping.

Figure 7 contains the results for varying frequency. The calculations indicate that as the frequency increases, the size of the oxide smoke has an increasingly important influence. From Eq. (1) it can be seen that for small particles, the

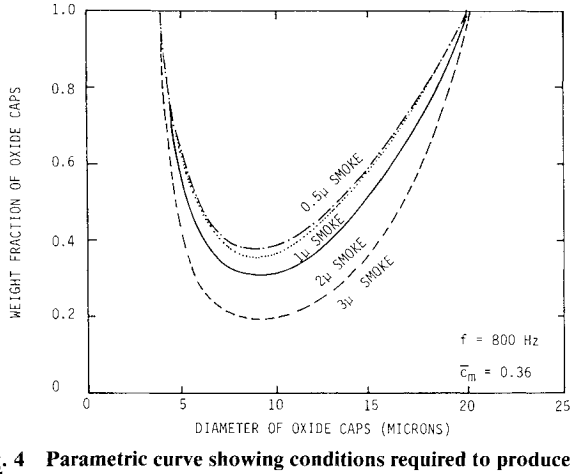

Fig. 4 Parametric curve showing conditions required to produce 125 s^{-1} of damping at 800 Hz corresponding to pulsed T-burner data.

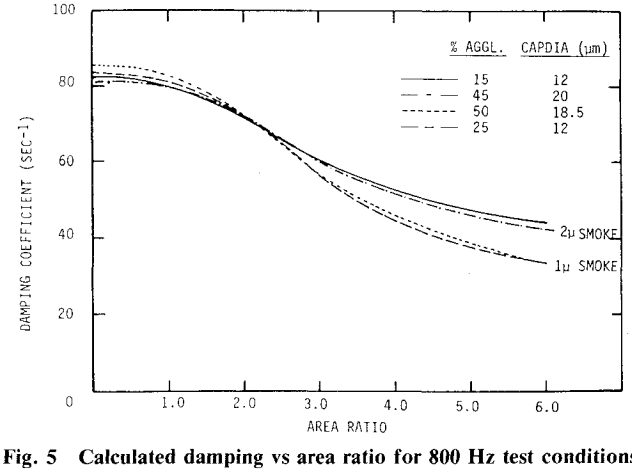

Fig. 5 Calculated damping vs area ratio for 800 Hz test conditions (cases 1-4).

Table 1 Parametric conditions

a) Postulated particulate conditions

Case	I	II	III	IV
Agglomerated particulate, %	50	25	15	45
Cap diameter, μm	18.5	12	12	20
Smoke diameter, μm	1.0	1.0	2.0	2.0

b) Experimental test parameters simulated

Case	1	2	3	4	5	6
Pressure, MPa	3.4	2.0	4.8	3.4	3.4	3.4
(psi)	(500)	(300)	(700)	(500)	(500)	(500)
Burn rate, cm/s	0.81	0.64	0.89	0.81	0.81	0.81
(in./s)	(0.32)	(0.25)	(0.35)	(0.32)	(0.32)	(0.32)
Frequency, Hz	800	800	800	300	1150	1600

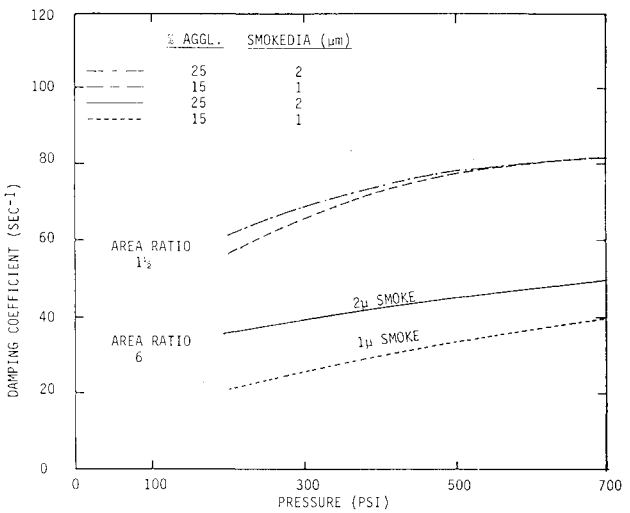

Fig. 6 Calculated damping vs pressure at 800 Hz for varying particle sizes and area ratios.

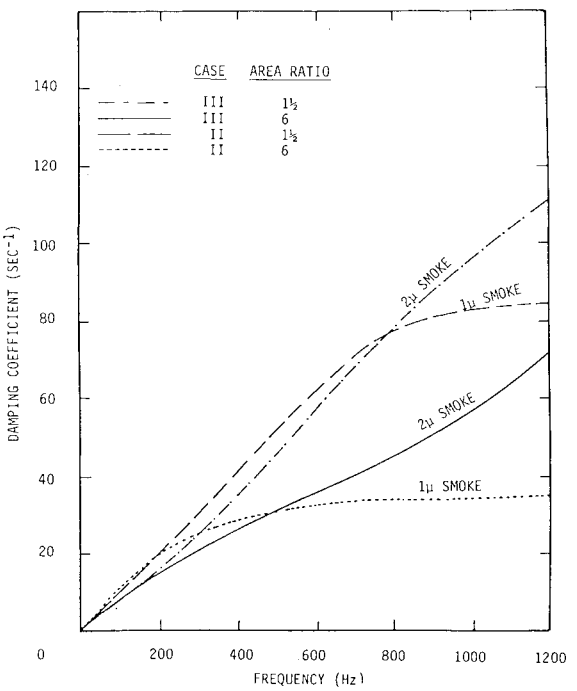

Fig. 7 Calculated damping vs frequency for varying particle sizes and area ratios.

damping increases as frequency squared. However, T-burner data[15] for aluminized propellants have shown a linear dependence of damping on frequency. Based on that data, Fig. 7 would seem to indicate that the assumed 2 μm smoke may be closer to the reported data (i.e., a linear dependence on frequency) than the calculations for 1 μm smoke. In either case, the calculated variation with area ratio (i.e., residence time) apparently reaches an asymptote at $\geq$800 Hz. For the shorter residence times the damping is due almost entirely to the smoke. The agglomerates are too large and do not have time to burn down to a size that would give effective damping.

Conclusions

A model has been developed to calculate the effect of distributed combustion on viscous particle damping due to an acoustic wave. The model accounts for the changing particle size of burning aluminum for both original aluminum particles and agglomerated aluminum. The aluminum burns directly to oxide smoke while the agglomerate burns to oxide smoke initially, ultimately forming an oxide cap of a specified size. The calculations have been compared to variable area T-burner data for a typical aluminized composite propellant. The calculated results show that significant amounts of the metal can leave the burner unburned, causing a change in damping for changing residence time. The results show that the assumption that particle damping is independent of propellant area ratio is not valid. The calculated particle damping changed by a factor of two over the typical range of conditions used in variable area T-burners. For actual motors the anticipated damping is significantly different from what would be calculated based on T-burner data.

Acknowledgment

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